A NEW PRODUCT GROWTH MODEL FOR CONSUMER DURABLES

– FRANK M. BASS

INSTITUTE FOR RESEARCH
IN THE BEHAVIORAL, ECONOMIC,
AND MANAGEMENT SCIENCES

INSTITUTE PAPER
NO. 175

HERMAN C. KRANNERT GRADUATE SCHOOL
of
INDUSTRIAL ADMINISTRATION
PURDUE UNIVERSITY

PURDUE UNIVERSITY

KRANNERT SCHOOL OF INDUSTRIAL ADMINISTRATION INSTITUTE PAPER SERIES

Copies of the following papers may be obtained by writing to The Editor, Institute Paper Series, School of Industrial Administration, Purdue University, Lafayette, Indiana. An asterisk (*) after the title indicates that the supply has been exhausted, though copies may occasionally be obtained by writing directly to the author. The symbol, #, indicates that the paper has been subsequently published, and is available in either the Institute Series or published version.

1964

65. Charles W. Howe, PROCESS AND PRODUCTION FUNCTIONS FOR INLAND WATERWAY TRANSPORTATION.*
66. Donald B. Rice, PRODUCT LINE SELECTION AND DISCRETE OPTIMIZING.*
67. William Starbuck, ORGANIZATIONAL GROWTH AND DEVELOPMENT.#*
68. Cliff Lloyd, ON THE FALSIFIABILITY OF TRADITIONAL DEMAND THEORY.#*
69. Vernon L. Smith, EXPERIMENTAL AUCTION MARKETS AND THE WALRASIAN HYPOTHESIS.#*
70. Yasusuke Murakami, BALANCED GROWTH UNDER EXOGENOUS LABOR GROWTH. *
71. Paul De Schutter, AN APPRAISAL OF A FEW EXAMPLES OF CONTEMPORARY ECONOMETRIC ANALYSIS.*
72. James P. Streamo, TESTING ECONOMETRIC MODELS.*
73. Karl E. Weick, LABORATORY EXPERIMENTATION WITH ORGANIZATIONS.*
74. James Quirk and Richard Ruppert, QUALITATIVE ECONOMICS AND THE STABILITY OF EQUILIBRIUM.#*
75. Vernon L. Smith, ON PRODUCTION FUNCTIONS OF CONSTANT ELASTICITY OF SUBSTITUTION.
76. Hugo Sonnenschein, THE RELATIONSHIP BETWEEN TRANSITIVE PREFERENCE AND THE STRUCTURE OF THE CHOICE SPACE.
77. Charles W. Howe, MODELS OF A BARGELINE: AN ANALYSIS OF RETURNS TO SCALE IN INLAND WATERWAY TRANSPORTATION.*
78. R. L. Basmann, ON PREDICTIVE TESTING OF A SIMULTANEOUS EQUATIONS MODEL: THE RETAIL MARKET FOR FOOD IN THE U.S.*
79. Thomas Joseph Muench, CONSISTENCY OF LEAST SQUARE ESTIMATORS OF COEFFICIENTS IN EXPLOSIVE STOCHASTIC DIFFERENCE EQUATIONS.*
80. Peter Jason Kalman, THEORY OF CHOICE WHEN PRICES ENTER THE UTILITY FUNCTION. *
81. Yasusuke Murakami, BALANCED GROWTH UNDER EXOGENOUS LABOR GROWTH: II*
82. George Horwich, AN INTEGRATED ANALYSIS OF AGGREGATE SUPPLY AND DEMAND.*
83. Peter Jason Kalman, A CLASS OF UTILITY FUNCTIONS ADMITTING TYRNI'S HOMOGENEOUS SAVING FUNCTION.
84. Peter Jason Kalman, PROFESSOR PEARCE'S ASSUMPTIONS AND THE NONEXISTENCE OF A UTILITY FUNCTION.
85. Richard E. Walton, THEORY OF CONFLICT IN LATERAL ORGANIZATIONAL RELATIONSHIPS.*
86. Richard E. Walton and Robert B. McKersie, ATTITUDE CHANGE IN INTERGROUP RELATIONS.*
87. William H. Starbuck, MATHEMATICS AND ORGANIZATION THEORY.#*
88. Peter Jason Kalman, THE EXISTENCE OF A GLOBALLY DIFFERENTIABLE DEMAND FUNCTION.
89. Vernon L. Smith, BIDDING THEORY AND THE TREASURY BILL AUCTION: DOES PRICE DISCRIMINATION INCREASE BILL PRICES?#
90. Yasusuke Murakami, FORMAL STRUCTURE OF MAJORITY DECISION.
91. Nancy Lou Schwartz, ECONOMIC TRANSPORTATION FLEET COMPOSITION AND SCHEDULING, WITH SPECIAL REFERENCE TO INLAND WATERWAY TRANSPORT.
92. J. M. Dutton and R. E. Walton, INTERDEPARTMENTAL CONFLICT AND COOPERATION: TWO CONTRASTING STUDIES.*
93. R. E. Walton, J. M. Dutton, H. G. Fitch, A STUDY OF CONFLICT IN THE PROCESS, STRUCTURE AND ATTITUDES OF LATERAL RELATIONSHIPS. #*
94. Edgar A. Pessemier, PRODUCT POLICY.#
95. Richard E. Walton, TWO STRATEGIES OF SOCIAL CHANGE AND THEIR DILEMMAS.#*
96. John J. Sherwood, SELF IDENTITY AND THE SOCIAL ENVIRONMENT.#*

1965

97. Michael J. Driver, A STRUCTURAL ANALYSIS OF AGGRESSION, STRESS, AND PERSONALITY IN AN INTER-NATION SIMULATION.
98. George Horwich, TIGHT MONEY, MONETARY RESTRAINT, AND THE PRICE LEVEL.#*
99. Vernon L. Smith, DISCRIMINATION VS. COMPETITION IN SEALED BID AUCTION MARKETS: A STUDY IN INDIVIDUAL AND MARKET BEHAVIOR.*
100. John J. Sherwood, AUTHORITARIANISM AND MORAL REALISM.#*
101. Keith V. Smith, CLASSIFICATION OF INVESTMENT SECURITIES USING MULTIPLE DISCRIMINANT ANALYSIS.
102. James Streamo, ANOTHER LOOK AT THE RETAIL FOOD MARKET IN THE UNITED STATES: 1942–1959 (TESTING AN ECONOMETRIC MODEL).
103. Yo Fukuba, DYNAMIC NETWORK FLOWS.
104. R. L. Basmann, ON THE EMPIRICAL TESTABILITY OF 'EXPLICIT CAUSAL CHAINS' AGAINST THE CLASS OF 'INTERDEPENDENT' MODELS. *

A NEW PRODUCT GROWTH MODEL FOR CONSUMER DURABLES

BY

FRANK M. BASS

PAPER NO. 175

JUNE 1967

INSTITUTE FOR RESEARCH
IN THE BEHAVIORAL, ECONOMIC
AND MANAGEMENT SCIENCES

HERMAN C. KRANNERT GRADUATE SCHOOL
OF
INDUSTRIAL ADMINISTRATION
PURDUE UNIVERSITY
LAFAYETTE, INDIANA

A New Product Growth Model For Consumer Durables*

Frank M. Bass

Krannert Graduate School of Industrial Administration

Purdue University

A growth model for the timing of initial purchase of new products is developed and tested empirically against data for eleven consumer durables. The basic assumption of the model is that the timing of a consumer's initial purchase is related to the number of previous buyers. A behavioral rationale for the model is offered in terms of innovative and imitative behavior. The model yields good predictions of the sales peak and the timing of the peak when applied to historical data. A long-range forecast is developed for the sales of color television sets.

The concern of this paper is the development of a theory of timing of initial purchase of new consumer products. The empirical aspects of the work presented here deal primarily with consumer durables.[1] The theory, however, is intended to apply to the growth of initial purchases of a broad range of distinctive "new" generic classes of products. Thus we draw a distinction between new classes of products as opposed to new brands or new models of older products.

Haines [1], Fourt and Woodlock [2], and others have suggested growth models for new brands or new products which suggests exponential growth to some asymptote. The growth model postulated here, however, is best reflected by growth patterns similar to that shown in Figure 1. Sales grow to a peak and then level off at some magnitude lower than the peak.

---

1 See the addendum for analysis of two non-durables.

* Some of the basic ideas in this paper were originally suggested to me by Peter Frevert, now of the University of Kansas. Thomas H. Bruhn, Gordon Constable, and Murray Silverman provided programming and computational assistance.

The stabilizing effect is accounted for by the relative growth of the replacement purchasing component of sales and the decline of the initial purchase component. We shall be concerned here only with the timing of initial purchase.

Figure 1
Growth of a New Product

Long-range forecasting of new product sales is a guessing game, at best. Some things, however, may be easier to guess than others. The theoretical framework presented here provides a rationale for long-range forecasting. The theory stems mathematically from the contagion models which have found such widespread application in epidemology. [3] Behaviorally, the assumptions are similar in certain respects to the theoretical concepts emerging in the literature on new product adoption and diffusion, [4], [5], [6] as well as to some learning models. [7], [8] The model differs from models based on the log-normal distribution [9] and other growth models in that the behavioral assumptions are explicit.

The Theory of Adoption and Diffusion

The theory of the adoption and diffusion of new ideas or new products by a social system has been discussed at length by Rogers. [4] This discussion is largely literary. It is therefore not always easy to separate the premises of the theory from the conclusions. In the discussion which follows an attempt will be made to outline the major ideas of the theory as they apply to the timing of adoption.

Some individuals decide to adopt an innovation independently of the decisions of other individuals in a social system. We shall refer to these individuals as innovators. We might ordinarily expect the first adopters to be innovators. In the literature, the following classes of adopters are specified: (1) Innovators, (2) Early Adopters, (3) Early Majority, (4) Late Majority, and (5) Laggards. This classification is based upon the timing of adoption by the various groups.

Apart from innovators, adopters are influenced in the timing of adoption by the pressures of the social system, the pressure increasing for later adopters with the number of previous adopters. In the mathematical formulation of the theory presented here we shall aggregate groups (2) through (5) above and define them as imitators. Imitators, unlike innovators, are influenced in the timing of adoption by the decisions of other members of the social system. Rogers defines innovators, rather arbitrarily, as the first two and one-half percent of the adopters. Innovators are described as being venturesome and daring. They also interact with other innovators. When we say that they are not influenced in the timing of purchase by other members of the social system, we mean that the pressure for adoption, for this group, does not increase with the growth of the adoption process. In fact, quite the opposite may be true.

In applying the theory to the timing of initial purchase of a new consumer product, we formulate the following precise and basic assumption which, hopefully, characterizes the literary theory: The probability that an initial purchase will be made at T given that no purchase has yet been made is a linear function of the number of previous buyers. Thus, $P(T) = p + \frac{q}{m} Y(T)$, where p and $\frac{q}{m}$ are constants and $Y(T)$ is the number of previous buyers. Since $Y(0) = 0$, the constant p is the probability of an initial purchase at $T = 0$ and its magnitude reflects the importance of innovators in the social system. Since the parameters of the model depend upon the scale used to measure time, it is possible to select a unit of measure for time such that p reflects the fraction of all adopters who are innovators in the sense in which Rogers defines them. The product $\frac{q}{m}$ times $Y(T)$ reflects the pressures operating on imitators as the number of previous buyers increases.

In the section which follows, the basic assumption of the theory will be formulated in terms of a continuous model and a density function of time to initial purchase. We shall therefore refer to the linear probability element as a likelihood.

Assumptions and the Model

The following fundamental assumptions characterize the model:

a) Over the period of interest there will be m initial purchases of the product.

b) The likelihood of purchase at time T given that no purchase has yet been made is $\frac{f(T)}{1-F(T)} = p + q\,F(T)$ where f(T) is the likelihood of purchase at T and $F(T) = \int_0^T f(t)\,dt$, and F(0) = 0. Therefore sales at T = $S(T) = mf(T) = \left[p + q \int_0^T \frac{S(t)}{m}\,dt\right]\left[m - \int_0^T S(t)\,dt\right]$.

The behavioral rationale for these assumptions are summarized:

a) Initial purchases of the product are made by "innovators" and "imitators," the important distinction between an innovator and an imitator being the buying motive. Innovators are not influenced in the timing of their initial purchase by the number of people who have already bought the product, while imitators are influenced by the number of previous buyers. Imitators "learn," in some sense, from those who have already bought.

b) The importance of innovators will be greater at first but will diminish monotonically with time.

c) We shall refer to p as the coefficient of innovation and q as the coefficient of imitation.

Since $f(T) = [p + q\,F(T)]\,[1 - F(T)] = p + (q - p)\,F(T) - q\,[F(T)]^2$, in order to find F(T) we must solve this non-linear differential equation: $dT = \frac{dF}{p + (q - p)\,F - qF^2}$. The solution is:

$$F = \frac{(q - pe^{-(T + C)(p + q)})}{q\,(1 + e^{-(T + C)(p + q)})}$$

Since $F(0) = 0$, the integration constant may be evaluated:

$$-C = \frac{1}{p + q} \text{Ln } (q/p) \text{ and } F(T) = \frac{(1 - e^{-(p + q) T})}{(q/pe^{-(p + q) T} + 1)}$$

$$f(T) = \frac{(p + q)^2}{p} \quad \frac{e^{-(p + q) T}}{(q/pe^{-(p + q) T} + 1)^2} \quad \text{, and}$$

To find the time at which the sales rate reaches its peak, we differentiate S,

$$S' = \frac{m/p\ (p + q)^3\ e^{-(p + q) T} (q/pe^{-(p + q) T} - 1)}{(q/pe^{-(p + q) T} + 1)^3}$$

Thus, $T^* = -\frac{1}{p + q} \text{Ln}(p/q) = \frac{1}{p + q} \text{Ln}(q/p)$ and if an interior maximum exists, $q > p$. The solution is depicted graphically in Figure 2 and 3.

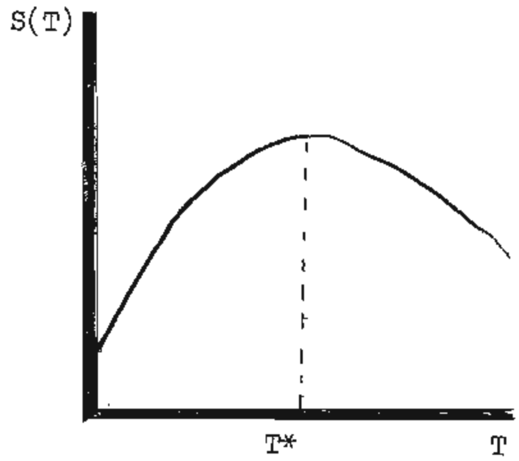

Figure 2
Growth Rate
$(q > p)$

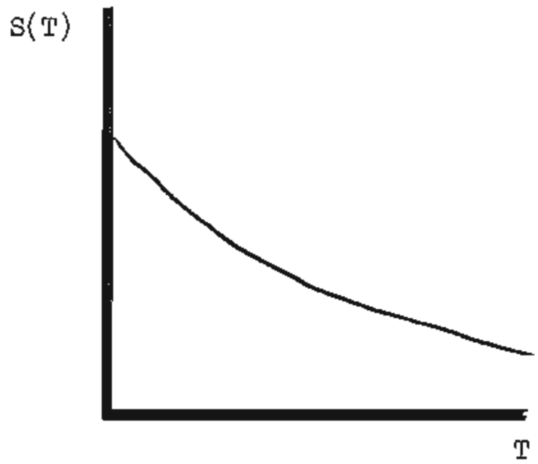

Figure 3
Growth Rate
$(q \leq p)$

We note that $S(T^*) = \frac{m(p + q)^2}{4q}$ and $Y(T^*) = \int_0^T S(t)\,dt = \frac{m(q - p)}{2q}$.

Since for successful new products the coefficient of imitation will ordinarily be much larger than the coefficient of innovation, sales is approximately one-half m. We note also that the expected time to purchase, E(T), is $\frac{1}{q}\,\mathrm{Ln}\left(\frac{p + q}{p}\right)$.

The Discrete Analogue

The basic model is: $S(T) = pm + (q - p)\,Y(T) - q/m\,Y^2(T)$.

In estimating the parameters p,q, and m from discrete time series data we use the following analogue: $S_T = a + bY_{T-1} + cY^2_{T-1}$, $T = 2,3\ldots$

where: S_T = sales at T, and $Y_{T-1} = \sum_{t=1}^{T-1} S_t$ = cumulative sales through period T-1. Since a estimates pm, b estimates q-p, and c estimates $-q/m$: $-mc = q$, $a/m = p$.

Then $q - p = -mc - a/m = b$, and $c\,m^2 + bm + a = 0$, or $m = \frac{-b \pm \sqrt{b^2 - 4ca}}{2c}$ and the parameters p, q, and m are identified. If we write $S(Y_{T-1})$ and differentiate with respect to Y_{T-1}, $\frac{dS_T}{d_{Y_{T-1}}} = b + 2cY_{T-1}$.

Setting this equal to 0, $Y^*_{T-1} = \frac{-b}{2c} = \frac{m(q - p)}{2q} = Y(T^*)$, and $S_T(Y^*_{T-1}) = a - \frac{b^2}{2c} + \frac{b^2}{4c} = \frac{m(p + q)^2}{4q} = S(T^*)$. Therefore, the maximum value of S as a function of time coincides with the maximum value of S as a function of cumulative sales.

Regression Analysis

In order to test the model, regression estimates of the parameters were developed using annual time series data for eleven different consumer durables. The period of analysis was restricted in every case to include only those intervals in which repeat purchasing was not a factor of importance. Table 1 displays the regression results.

The data appear to be in good agreement with the model. The R^2 values are reasonably high and the parameter estimates seem reasonable for the model. Figures 4, 5, and 6 show the actual values of sales and the values predicted by the regression equation for three of the products analyzed. For every product studied the regression equation describes the general trend of the time path of growth very well. In addition, the regression equation provides a very good fit with respect to both the magnitude and the timing of the peaks for all of the products. Deviations from trend are largely explainable in terms of short-term income variations. This is especially apparent in Figure 5, where it is easy to identify recessions and booms in the years of sharp deviations from trend.

Table I

Growth Model Regression Results For Eleven Consumer Durable Products

Product	Period Covered	$\hat{a}$ (10^3)	$\hat{b}$	$\hat{c}$ (10^{-7})	R^2	$\frac{\hat{a}}{\sigma_{\hat{a}}}$	$\frac{\hat{b}}{\sigma_{\hat{b}}}$	$\frac{\hat{c}}{\sigma_{\hat{c}}}$	m (10^3)	p	q
Electric Refrigerators	1920-1940	104.67	.21305	-.05[illegible]13	.903	1.164	6.142	-2.548	40,001	.0026167	.21566
Home Freezers	1946-1961	308.12	.15298	-.077868	.742	4.195	4.769	-3.619	21,973	.018119	.17110
Black and White Television	1946-1961	2,696.2	.22317	-.025957	.576	3.312	3.724	-3.167	96,717	.027877	.25105
Water Softeners	1949-1961	.10256	.27925	-512.59	.919	3.593	8.089	-6.451	5,793	.017703	.29695
Room Air Conditioners	1946-1961	175.69	.40820	-.24777	.911	1.915	8.317	-6.034	16,895	.010399	.41861
Clothes Dryer	1948-1961	259.67	.33968	-.23647	.896	2.941	7.427	-5.701	15,092	.017206	.35688
Power Lawnmowers	1948-1961	410.98	.32871	-.075506	.932	1.935	7.408	-4.740	44,751	.0091837	.33790
Electric Bed Coverings	1949-1961	450.04	.23800	-.031842	.976	3.5[illegible]	6.820	-1.826	76,589	.005876	.24387
Automatic Coffee Makers	1948-1961	1,008.2	.28435	-.051242	.883	3.109	6.186	-4.353	58,838	.017135	.30145
Steam Irons	1949-1960	1,594.7	.29928	-.058875	.828	3.649	5.288	-4.318	55,696	.028632	.32791
Record Players	1952-1961	543.94	.62931	-.29817	.899	1.911	5.194	-3.718	21,937	.024796	.65410

Data Sources: Economic Almanac, Statistical Abstracts of the U.S., Electrical Merchandising, and Electrical Merchandising Week.

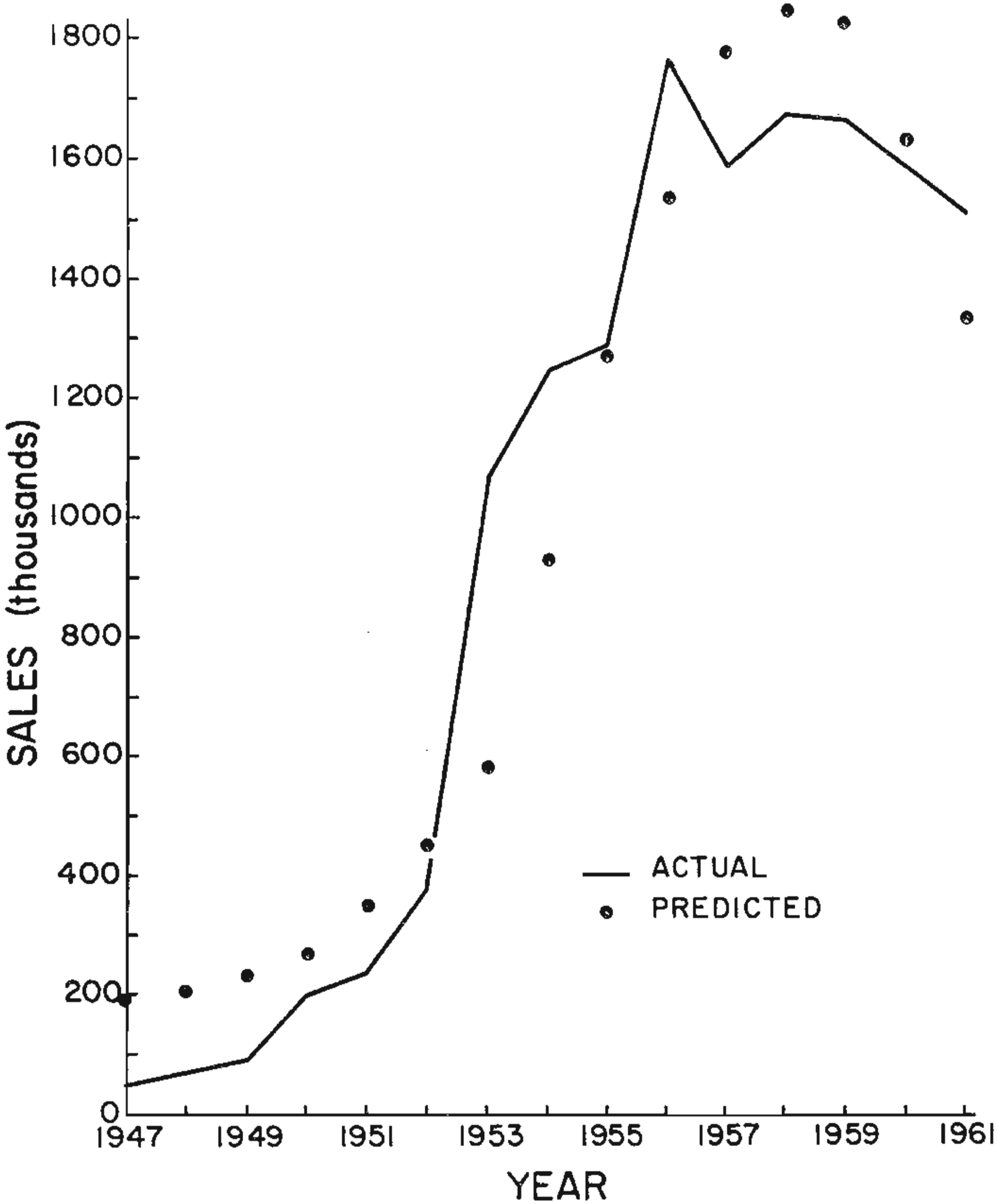

Figure 4 Actual Sales and Sales Predicted by Regression Equation (Room Air Conditioners)

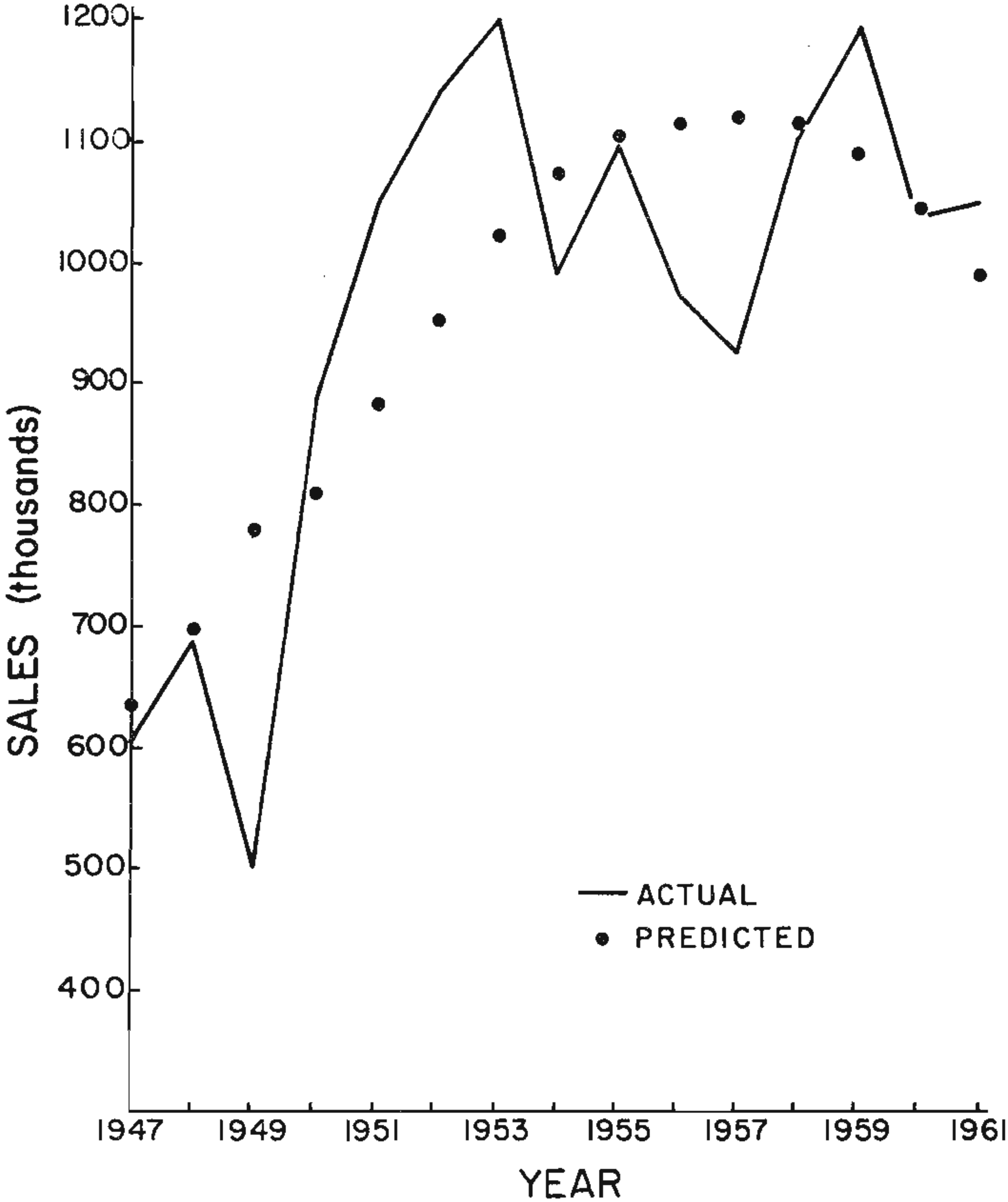

Figure 5 Actual Sales and Sales Predicted by Regression Equation (Home Freezers)

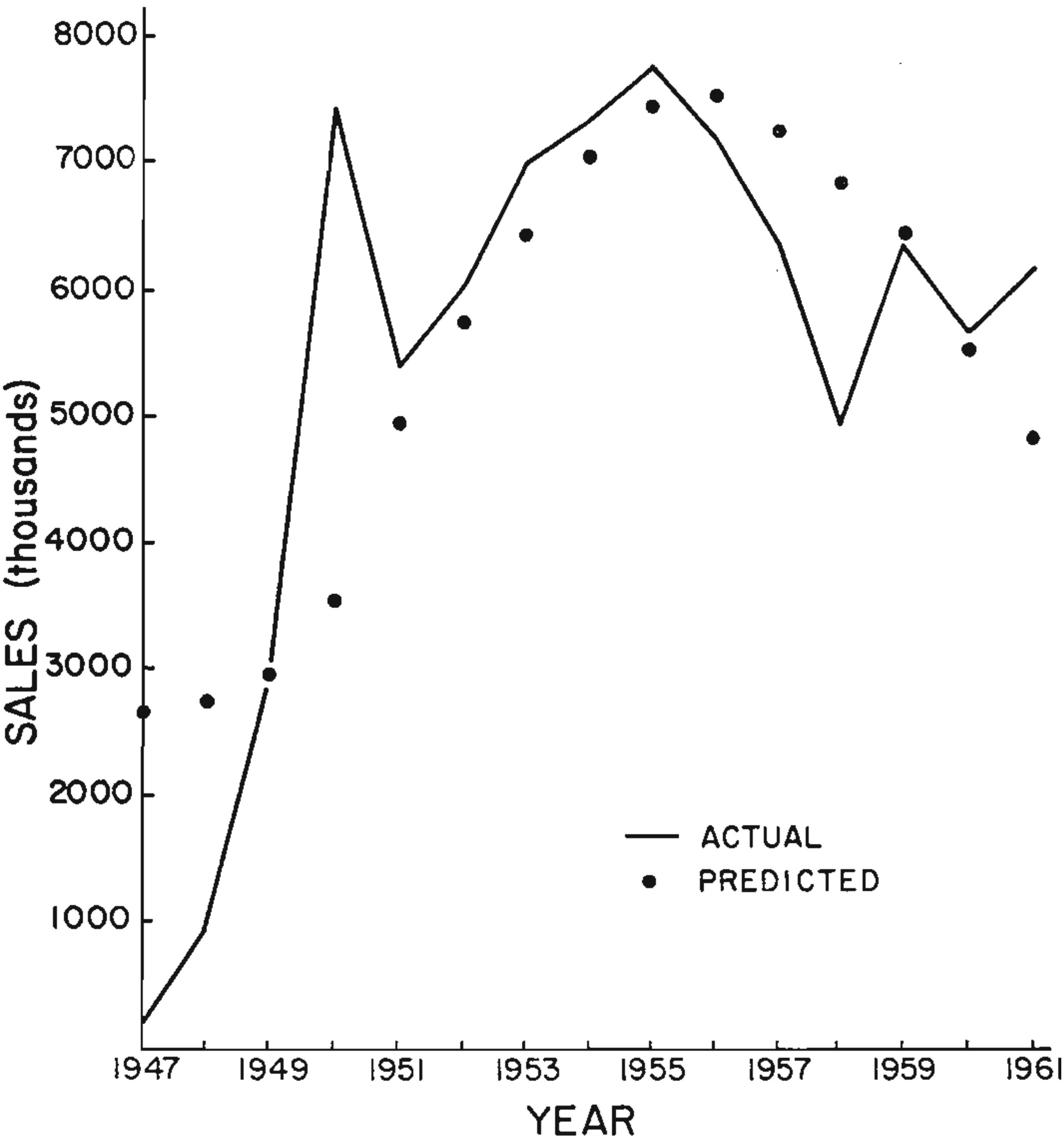

Figure 6 Actual Sales and Sales Predicted by Regression Equation (Black & White Television)

Model Performance

The performance of the regression equation relative to actual sales is a relatively weak test of the model's performance since it amounts to an ex post comparison of the regression equation estimates with the data. A much stronger test is the performance of the basic model with time as the variable and controlling parameter values as determined from the regression estimates. Table 2 provides a comparison of the model's prediction of time of peak and magnitude of peak for the eleven products studied.

Since, according to the model S(0) = pm, we identify time period 1 as that period in which sales equal or exceed $\hat{p}\hat{m}$ for the first time. It is clear from the comparison shown in Table 2 that the model provides good predictions of the timing and magnitude of the peaks for all eleven products studied.

In order to determine the accuracy with which it would have been possible to "forecast" period sales over a long-range interval with prior knowledge of the parameter values, the regression estimates of the parameters were substituted in the basic model,

$$S(T) = \frac{m(p + q)^2}{p} \quad \frac{e^{-(p + q)T}}{(q/pe^{-(p + q)T} + 1)^2},$$

and sales estimates generated for each of the products for each year indicated in the intervals shown in Table 3. In most cases the model provides a good fit to the data. Even in the few instances of low r^2 values, the model provides a good description of the general trend of the sales curve, the deviations from trend being sharp, but ephemeral. Figures 7, 8, and 9 illustrate the predicted and actual sales curves for three of the products.

Table 2

Comparison of Predicted Time and Magnitude of Peak with Actual Values for Eleven Consumer Durable Products

Product	q/p	Predicted Time of Peak $T^* = \frac{1}{p+q} \ln(q/p)$	Actual Time of Peak*	Predicted Magnitude of Peak $S(T^*) = \frac{m(p+q)^2}{4q}$ (10^6)	Actual Magnitude of Peak (10^6)
Electric Refrigerators	82.4	20.1	**	2.20	**
Home Freezers	9.4	11.6	13	1.2	1.2
Black & White Television	9.0	7.8	7	7.5	7.8
Water Softeners	16.7	8.9	9	.5	.5
Room Air Conditioners	40.2	8.6	7	1.8	1.7
Clothes Dryer	20.7	8.1	7	1.5	1.5
Power Lawnmowers	36.7	10.3	11	4.0	4.2
Electric Bed Coverings	41.6	14.9	14	4.8	4.5
Automatic Coffee Makers	18.1	9.0	10	4.8	4.9
Steam Irons	11.4	6.8	7	5.5	5.9
Record Players	26.3	4.8	5	3.8	3.7

*Time period one is defined as that period for which sales equal or exceed $\hat{p}\,\hat{m}$ for the first time.

**Interrupted by war. Prewar peak in year 16 (1940) at 2.6 x 10^6 units.

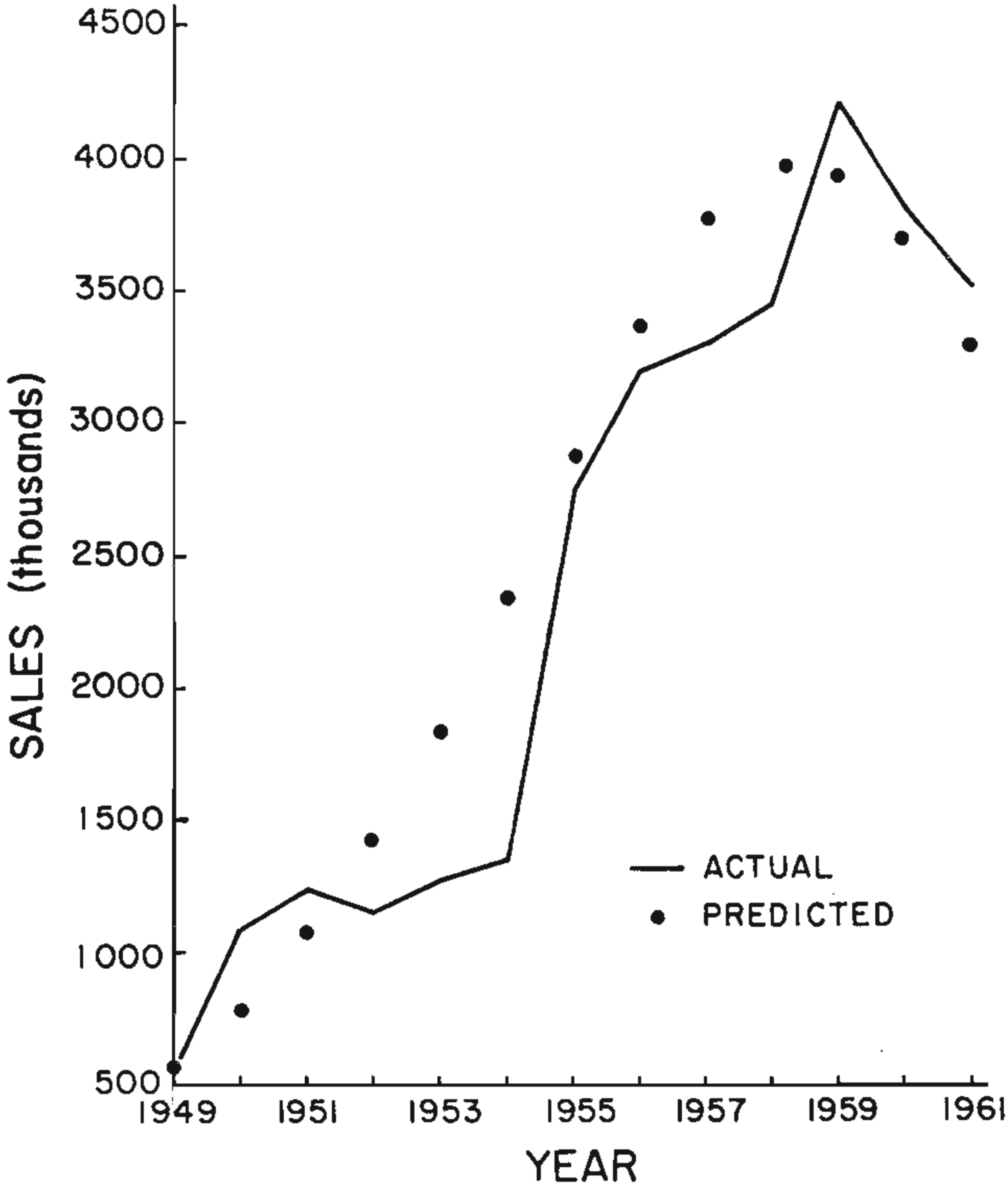

Figure 7 Actual Sales and Sales Predicted by Model (Power Lawnmowers)

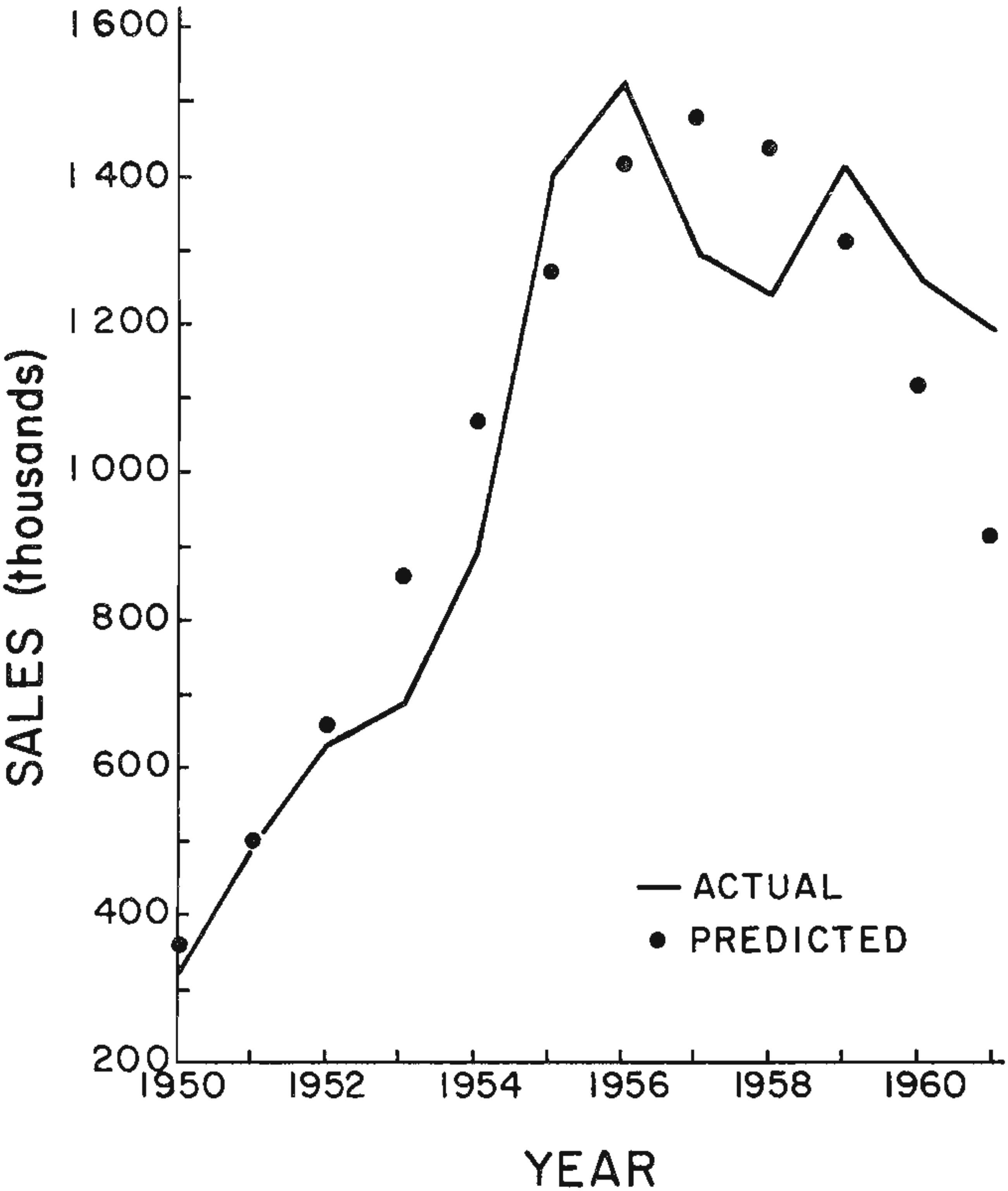

Figure 8 Actual Sales and Sales Predicted by Model (Clothes Dryers)

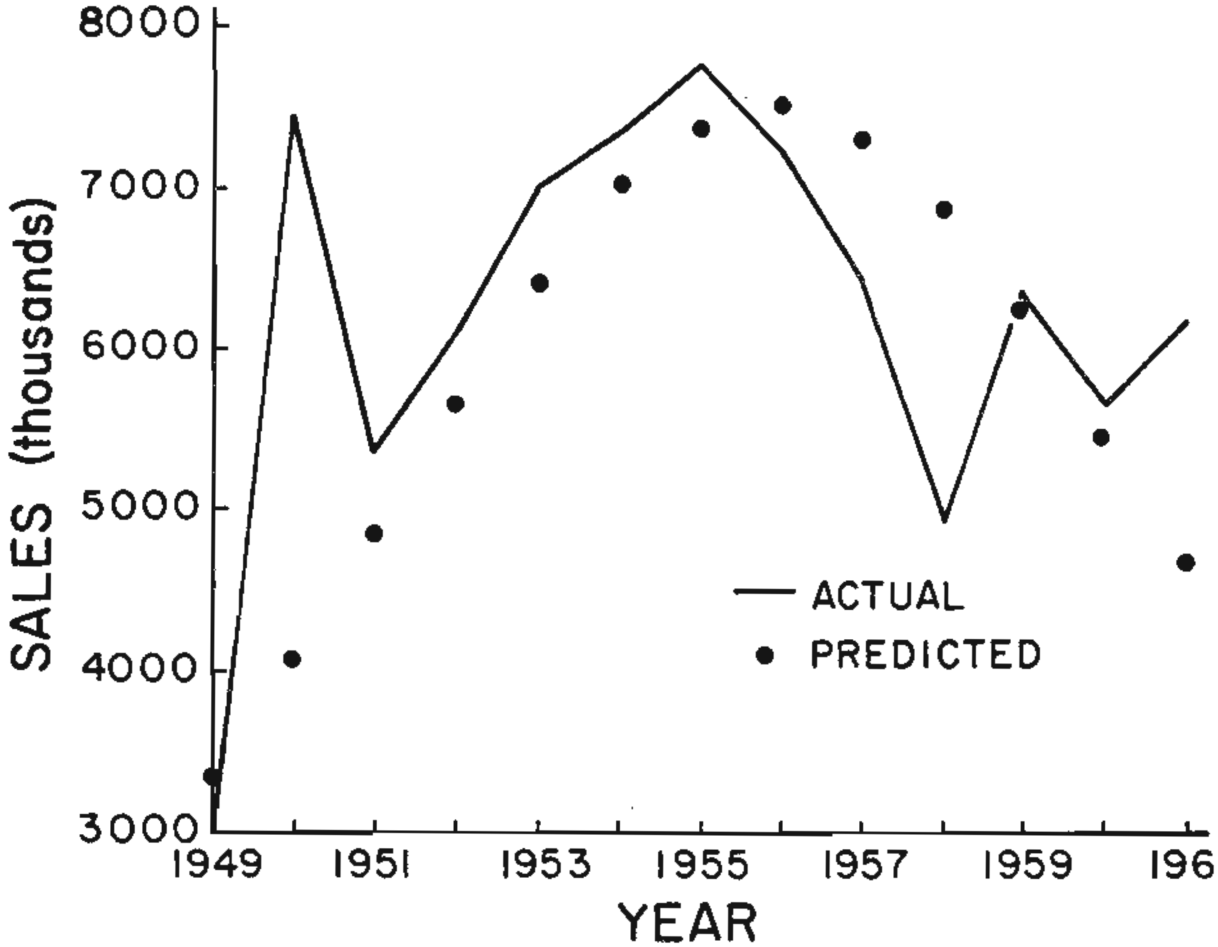

Figure 9 Actual Sales and Sales Predicted by Model
(Black & White Television)

It would appear fair to conclude that the data are in generally good agreement with the model. The model has, then, in some sense, been "tested" and verified. We may now claim to know something about the phenomenon we set out to explore. The question is, however, will this knowledge be useful for purposes of long-range forecasting?

Long-Range Forecasting

There are two cases worth considering in long-range forecasting: the no-data case and the limited-data case. For either of these possibilities one may well ask: is it easier to guess the sales curve for the new product or easier to guess the parameters of the model? No attempt will be made here to answer this question, in general, but it does seem likely that for some products it would be possible to make plausible guesses of the parameters. Analysis of the potential market and the buying motives should make it possible to guess at m, the size of the market, and of the relative values of p and q, the latter guess being determined by a consideration of buying motives. If the sales curve is to be determined by means other than the model suggested in this paper, the implications of this forecast in terms of the parameters of the model might be useful as a test of the credibility of the forecast.

In order to illustrate the forecasting possibilities in the limited data case, we shall develop a forecast for color television set sales. In principle, since there are three parameters to be estimated, some kind of estimate is possible with only three observations if the first of these observations occurs at $T = 0$. Any such estimate should be viewed with some skepticism, however, since the parameter estimates are very sensitive to small variations in the three observations. Before applying estimates obtained from

Table 3

Forecasting Accuracy of the Model for Eleven Consumer Durable Products

Product	Period of Forecast	r^2
Electric Refrigerators	1926-1940	.762
Home Freezers	1947-1961	.473
Black & White Television	1949-1961	.077*
Water Softeners	1950-1961	.920
Room Air Conditioners	1950-1961	.900
Clothes Dryers	1950-1961	.858
Power Lawnmowers	1949-1961	.898
Electric Bed Coverings	1950-1961	.934
Automatic Coffee Makers	1951-1961	.690
Steam Irons	1950-1961	.730
Record Players	1953-1958	.953

*The low "explained" variance for this product is accounted for by extreme deviation from trend in two periods. Actually, the model provides a fairly good description of the growth rate, as indicated in Figure 9.

a limited number of observations, the plausibility of these estimates should be closely scrutinized.

In substituting $\sum_{t=0}^{T-1} S_t$ in the discrete analogue for $\int_0^T S(t)\,dt$ in the continuous model, a certain bias was introduced. This bias is mitigated when there are several observations, but can be crucial when there are only a few. Thus, the proper formulation of the discrete model, if $S_T = S(T)$ is: $S_T = a + bk(T)\,Y_{T-1} + ck^2(T)\,Y^2_{T-1}$, where $k(T) = \frac{Y(T)}{Y_{T-1}}$. We note that for any probability distribution for which: a) $f(x) = 1/k\,[F(x+1) - F(x)]$, and b) $F(0) = 0$, $\sum_{t=0}^{x-1} f(t) = 1/k\,F(x)$. In particular, these two properties hold for the exponential distribution. Therefore, for this distribution $\sum_{t=0}^{x-1} \frac{F(x)}{f(t)} = k$. The density function $f(T)$ in the growth model developed in this paper is approximately exponential in character <u>when p and T are small</u>. Thus, $f_{apx}(T) = \frac{1}{k}\,[F_{apx}(T+1) - F_{apx}(T)]$ and $1/k = \frac{(p+q)}{[e^{(p+q)} - 1]}$. For small values of T we therefore write: $S_T = a + b'Y_{T-1} + c'Y^2_{T-1}$, where $b' = kb$, and $c' = k^2c$. Then $m = 1/km'$, $q = kq'$ and $p = kp'$. The value of $1/k$ for each of several different values of p + q has been calculated and appears in Table 4.

$m = k\,m'$
$q = \frac{1}{k}q'$
$p = \frac{1}{k}p'$

if $g = \frac{1}{k_2} g'$

$\frac{1}{k_2} = .97 - .4(p+g)$

~~then~~

$$g = \frac{.97 g'}{1 + .4(1 + 1/\alpha) g'} = \frac{1}{k_2} g'$$

$$\left[\frac{1}{k_2} = \frac{(p+g)}{[e^{(p+g)} - 1]} \right]$$

$m = k_2 m'$

$g = \frac{1}{k_2} m'$

$p = \frac{1}{k_2} p'$

$$\frac{1}{k_2} = \frac{.97}{1 + .4(1 + \frac{1}{\alpha}) g'}$$

$a = a'$

$b = \frac{1}{k_2} b'$

$\frac{1}{k_2} \cdot 8$

$c = \frac{1}{k_2^2} c'$

$$\frac{(p+q)^2}{p} \frac{e^{-(p+q)T}}{[q/p\, e^{-(p+q)T}+1]^2}$$

$$(p+q)^2 \frac{p}{q^2} e^{(p+q)T}$$

$$F(T) = (p+q)^2 \frac{p}{q^2} \int e^{(p+q)T} = (p+q) \frac{p}{q^2} e^{(p+q)T}$$
$$= \frac{1}{p+q} f(T)$$

$$f(T) = (p+q) F(T)$$
$$f(T) = (p+q)\, e^{-(p+q)} F(T+1)$$
$$F(T+1) - F(T) = \frac{1}{(p+q)} f(T) \left[e^{(p+q)}, 1 \right]$$
$$= k\, f(T)$$

$$f(T) = \frac{(p+q)}{\left[e^{(p+q)}\; 1\right]} \left[F(T+1) - F(T)\right]$$

$$m' = \frac{-b' - \sqrt{b'^2 - 4ac'}}{2c'} = \frac{1}{k} m$$

$$\boxed{m = k\, m'}$$
$$\boxed{q = \tfrac{1}{k} q'}$$
$$\boxed{p = \tfrac{1}{k} p'}$$

$$q' = k\, q$$
$$= .97q - .4\left(1 + \tfrac{1}{\theta}\right) q^2$$

$$.4\left(1+\tfrac{1}{\theta}\right) q^2 - .97q + q' = 0$$

$$q = \frac{.97 \pm \sqrt{(.97)^2 - 1.6\left(1+\tfrac{1}{\theta}\right) q'}}{.8\left(1+\tfrac{1}{\theta}\right)}$$

Table 4

Calculated Values of $1/k$ and $(p + q)$

$(p + q)$	$1/k = \frac{(p + q)}{[e^{(p + q)} - 1]}$
.3	.85
.4	.81
.5	.77
.6	.73
.7	.69
.8	.65
.9	.61

On the basis of the relationship between $1/k$ and $(p + q)$ indicated in Table 4: $1/k = .97 - .4\ (p + q)$, $q = \frac{1}{k}q' = \frac{.97\ q'}{1 + .4\ (1 + 1/\theta)\ q'}$, where $\theta = q'/p' = q/p$, and $p = \frac{p'}{q'}\,q = \frac{.97\ p'}{1 + .4\ (1 + \theta)p'}$.

We turn now to the forecast of color television set sales. The following data are available:

Sales (Millions of Units)	Year
.7	1963
1.35	1964
2.50	1965

Solving the following system of equations:

$$S_0 = .7 = a$$

$$S_1 = 1.35 = a + .7\ b' + .49\ c'$$

$$S_2 = 2.50 = a + 2.05\ b' + 4.20\ c', \text{ we find:}$$

$$a' = .7,\ b' = .954,\ c' = -.0374,$$

$$m' = 26.2,\ q' = .96,\ p' = .0267,$$

$$q = .67,\ p = .018,\ m = 37.4.$$

Table 5

Forecast of Color Television Sales 1966-1970

Year	Sales (millions)
1964	1.35
1965	2.5
1966	4.1
1967	5.8
1968	6.7
1969	6.3
1970	4.7

Since these parameter values appear plausible, they have been used in the basic model to generate the series of estimates of sales shown in Table 5 and Figure 10. The projected peak occurs in 1968 at around 7 million units. This forecast differs somewhat from some industry forecasts. At this writing, one company's research department has estimated that sales will "top out" in 1967 at between 7 and 8 million units. The forecast speaks for itself and the ultimate reality of actual sales and one's personal criterion of "goodness" will determine whether or not the forecast was a good one.

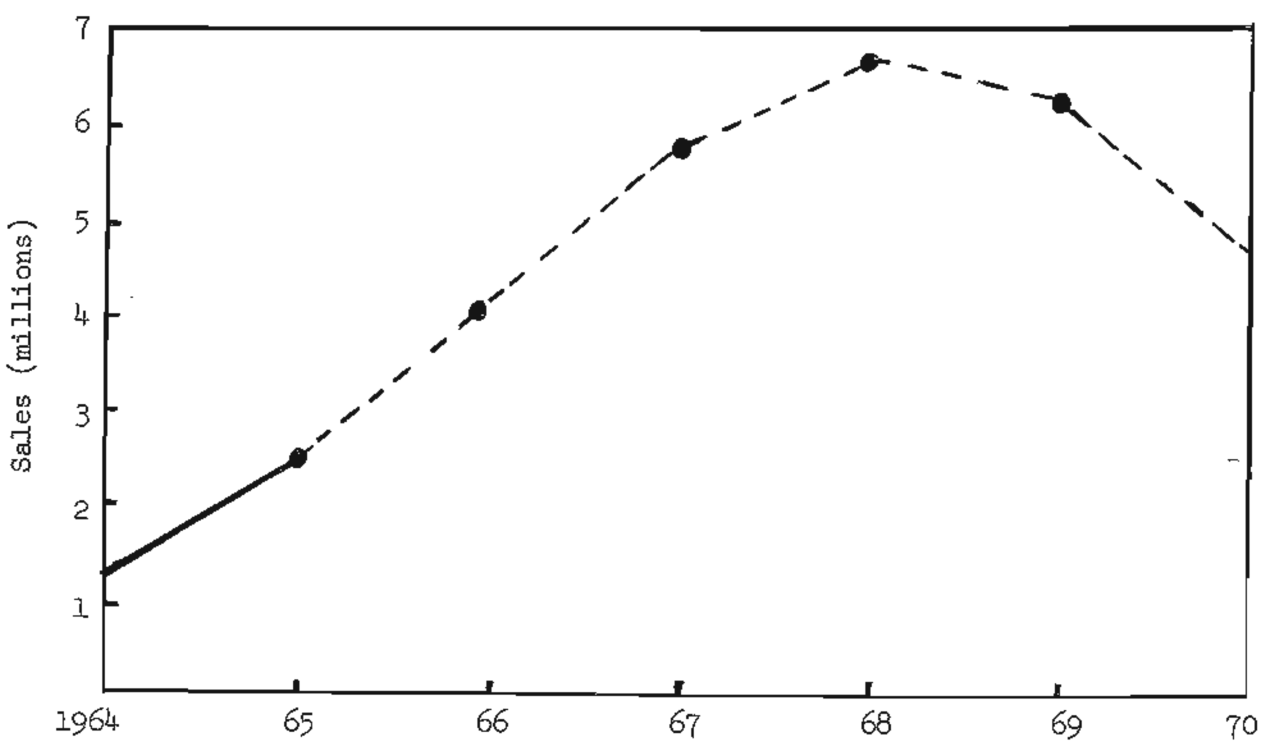

Figure 10
Projected Sales-Color Television

While this forecast was objectively determined in the sense that it was derived from data, it is also based upon a subjective judgment of the plausibility of the parameters. Since the parameter estimates are very sensitive to small variations in the observations when there are only a few observations, the importance of the plausibility test cannot be overemphasized.

Conclusion

The growth model developed in this paper for the timing of initial purchase of new products is based upon an assumption that the probability of purchase at any time is related linearly to the number of previous buyers. There is a behavioral rationale for this assumption. The model implies exponential growth of initial purchases to a peak and then exponential decay. In this respect it differs from other new product growth models.

Data for consumer durables are in good agreement with the model. Parameter estimates derived from regression analysis when used in conjunction with the model provide good descriptions of the growth of sales. From a planning viewpoint, probably the central interest in long-range forecasting lies in predictions of the timing and magnitude of the sales peak. The model provides good predictions of both of these variables for the products to which it has been applied. Insofar as the model contributes to an understanding of the process of new product adoption, the model may be useful in providing a rationale for long-range forecasting.

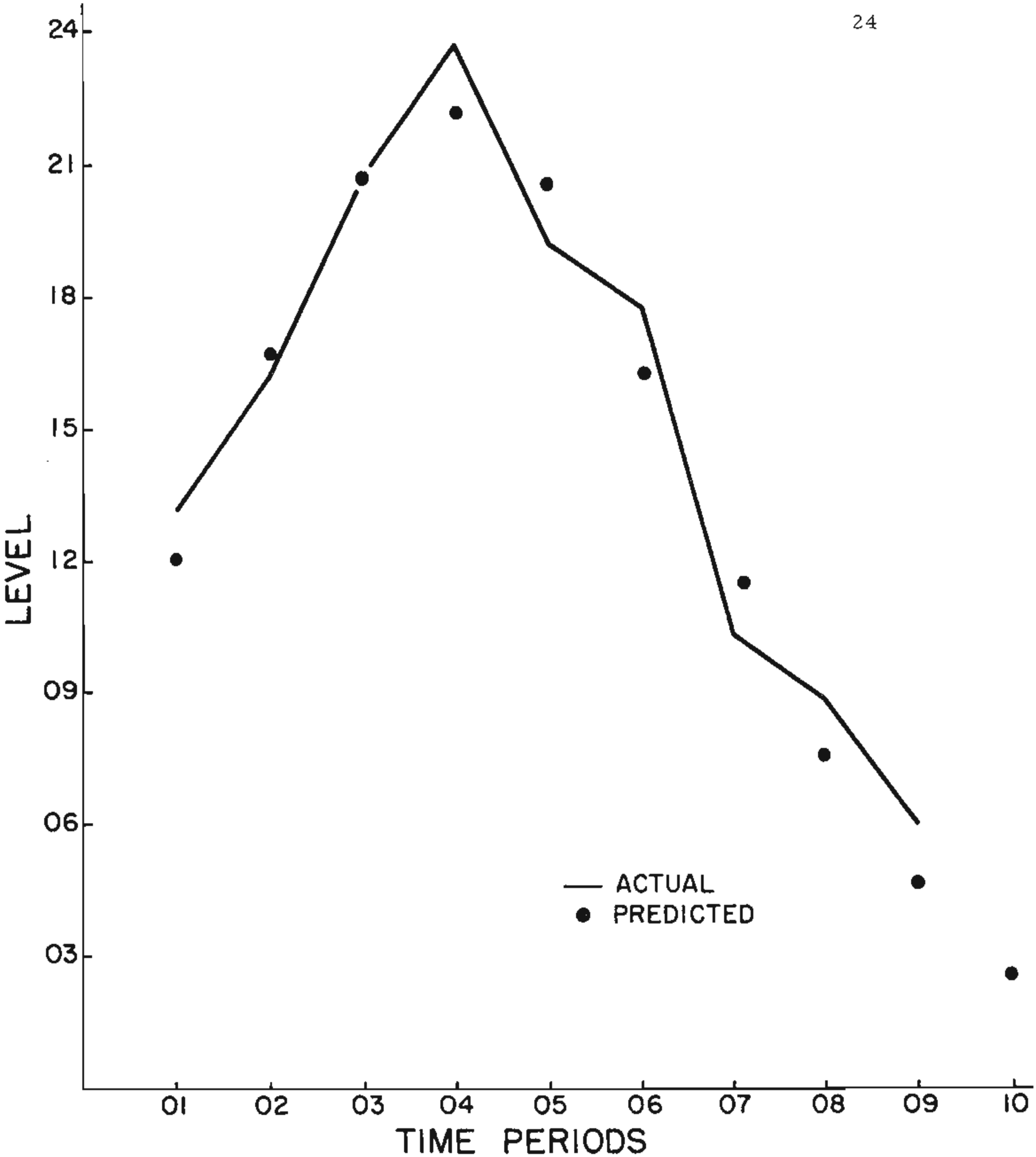

Figure 11 Actual Number Adopting and Number Predicted by Model (Weed Spray)

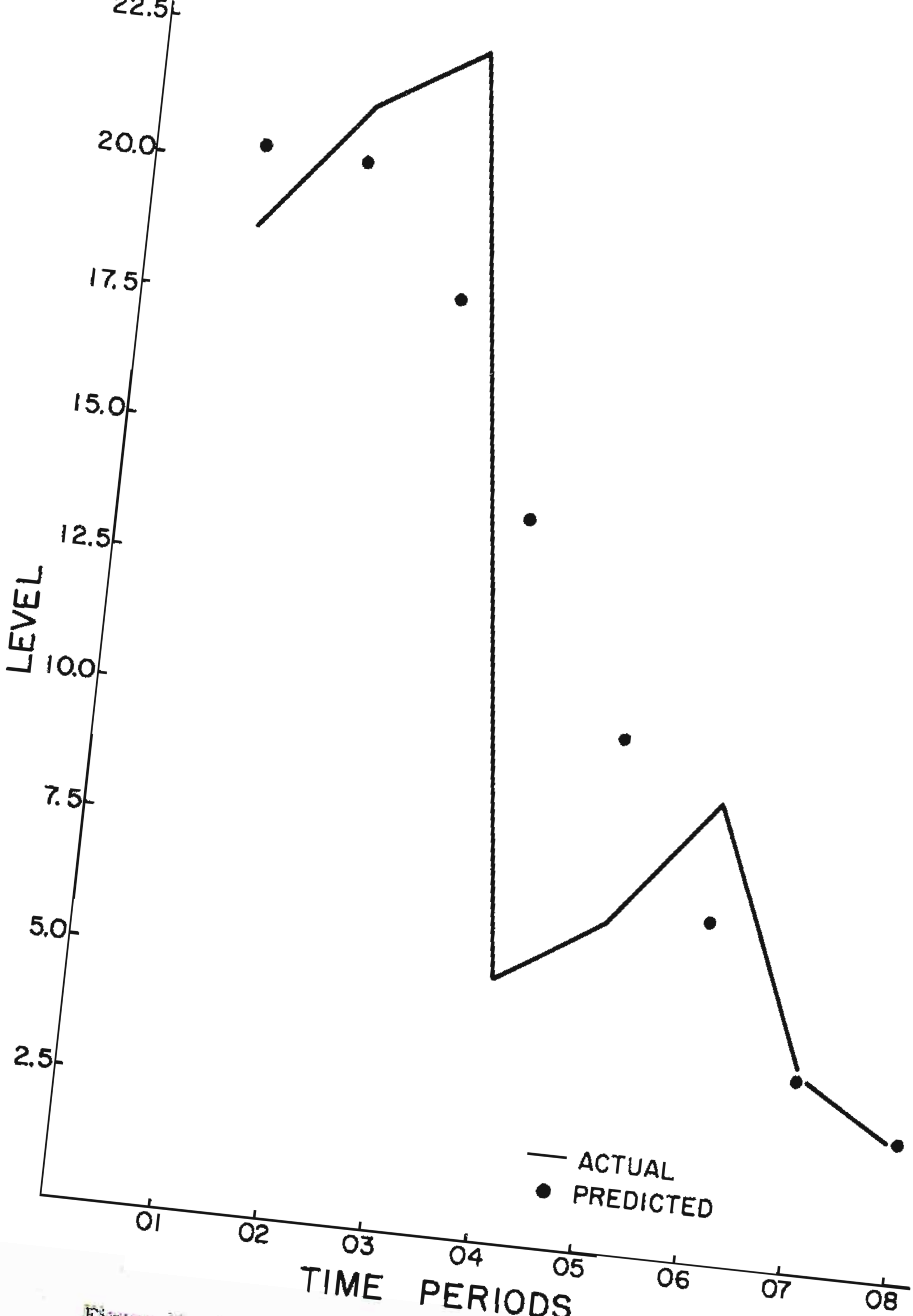

Figure 12 Actual Number Adopting and Number Predicted by Model (New Drug)

ADDENDUM

Survey Data, Non-Appliance Adoption

The adoption patterns for applicances analyzed in the main body of the paper were inferred from sales data on the premise that repeat purchasing was not a significant component of sales during the time period covered by the analysis. It is a matter of some interest, therefore, to examine the dynamics of the adoption process for additional product classes with non-sales data sources.

Published data based on survey and panel methods are available for two products--a weed spray and a new drug. Information was obtained from farmers on the timing of adoption of the weed spray and from physicians on the timing of adoption of the new drug. The data are shown below in Table 6. These data were obtained by reading from cumulative distribution graphs and therefore are slightly inaccurate.

Table 6

Adoption Data for Two New Products

Time Period	2,4-D Weed Spray Number Adopting	New Drug Number Adopting
1	13.32	18.75
2	16.28	21.25
3	20.72	22.50
4	23.68	5.00
5	19.24	6.25
6	17.76	8.75
7	10.36	3.75
8	8.88	2.50
9	5.92	

Sources: Rogers, E. M., Diffusion of Innovations (New York: The Free Press, 1962). p. 109.
Coleman, James, Menzel, Herbert, and Katz, Elihu, "Social Processes in Physicians Adoption of a New Drug," in Frank, R. E. Kuehn, A.A., and Massy, W. F., Quantitative Techniques in Marketing Analysis (Homewood: Richard D. Irwin, 1962) p. 241

The results of the regression analysis are summarized in Table 7 and Figures 11 and 12. The density function of time to initial purchase is again unimodel and the model adequately describes the data.

Table 7

Parameter Estimates For a Weed Spray and a New Drug

Parameters	Weed Spray	New Drug
a	8.04387	17.81431
b	.44472	.189229
c	-.00346	-.003277
m	144.1	107.07
q	.4998	.35086
p_2	.0558	.16638
R^2	.953	.827
r^2	.958	.791

References

1. Haines, G. H., Jr., "A Theory of Market Behavior After Innovation," Management Science, No. 4, Vol. 10, July, 1964.

2. Fourt, L. A. and Woodlock, J. W., "Early Prediction of Market Success for New Grocery Products," Journal of Marketing, No. 2, Vol. 26, October, 1960.

3. Bartlett, M. S., Stochastic Population Models in Ecology and Epidemiology, London: Methuen, 1960.

4. Rogers, E. M., Diffusion of Innovations, New York: The Free Press, 1962.

5. King, C. W., "Adoption and Diffusion Research in Marketing: An Overview," in Science, Technology and Marketing, 1966 Fall Conference of the American Marketing Association, Chicago, 1966.

6. Katz, E. and Lazarsfeld, F., Personal Influence, New York: The Free Press, 1955.

7. Rashevsky, N., Mathematical Biology of Social Behavior, Chicago: The University Press, 1959

8. Bush, R. R. and Mosteller, F., Stochastic Models for Learning, New York: Wiley, 1955.

9. Bain, A. D., The Growth of Television Ownership in the United Kingdom, A Lognormal Model, Cambridge: The University Press, 1964.

10. Dernburg, T. F., "Consumer Response to Innovation: Television," in Studies in Household Economic Behavior, Yale Studies in Economics, Vol. 9, New Haven, Connecticut: Yale, 1958.

11. Massy, W. F., "Innovation and Market Penetration," Ph.D. Thesis, Massachusetts Institute of Technology, 1960, Cambridge, Massachusetts.

12. Mansfield, E., "Technological Change and the Rate of Imitation," Econometrica, No. 4, Vol. 29, October, 1961.

13. Pessemier, E. A., New Product Decisions, An Analytic Approach, New York: McGraw-Hill, 1966.

A P P E N D I X

```
C        SALES PREDICTION ANALYSIS-REGRESSION COEFFICIENTS
         DIMENSION TITLE (80),S(50),IDNUM(50),ACTSAL(50)
         DATA DOLLAR, /1H$,1Hb /
       1 READ(5,100) TITLE
         WRITE(6,101) TITLE
         SENT=BLANK
         READ(5,102) A,B,C,N
         O=(-B-SQRT(B**2-4.*A*C))/(2.* C)
         P=A/O
         Q=-O*C
         K=1
         MM=0
         DO 7  I=1,N
         T=FLOAT(I)
         S(K)=(O*(P+Q)*(EXP((-P-Q*T)/((Q/P*EXP((-P-Q)*T)+1.)**2))
         IF(SENT.EQ.DOLLAR)  GO TO 6
         READ(5,103) IDNUM(K),ACTSAL(K),SENT
         MM=K
       6 IF(K.EQ.1.AND.ACTSAL(K).LT.A) GO TO 7
         K=K+1
       7 CONTINUE
         WRITE(6,104) O,P,Q
         WRITE(6,105)
         DO 8 I=1,MM
         WRITE(6,106)  I,IDNUM(I),S(I),ACTSAL(I)
       8 CONTINUE
         NN=MM+1
         IDNUMP=IDNUM(MM)
         K=K-1
         DO 9  I=NN,K
         IDNUMP=IDNUMP+1
         WRITE(6,107) I,IDNUMP,S(I)
       9 CONTINUE
         SUMSQD=0.0
         SUM=0.0
         SUMSQ=0.0
         DO  10 I=1,MM
         SUMSQ=SUMSQ+(S(I)-ACTSAL(I))**2
      10 SUM=SUM+ACTSAL(I)
         DO 11 I=1,MM
      11 SUMSQD=SUMSQD+(ACTSAL(I)-(SUM/FLOAT(MM)))**2
         RSQ=1.-(SUMSQ/SUMSQD)
         WRITE(6,108) RSQ
         GO TO 1
     100 FORMAT(80A1)
     101 FORMAT(1H1,19H PRODUCT ANALYZED. , 80A1)
     102 FORMAT(3F20.8,I2)
     103 FORMAT(I4,F10.3,65X,A1)
     104 FORMAT(1H ,13H COEFFICIENTS,10X,4H M= ,F10.3,10X,4H P= ,F12.8,10X,
        14H Q= ,F12.8///)
     105 FORMAT(1H ,13H SALES PERIOD,10X,5H YEAR,10X,10H EST-SALES,10X,10H
        1ACT-SALES)
     106 FORMAT(1H ,11X,I2,11X,I4,11X,F9.3,11X,F9.3)
     107 FORMAT(1H ,11X,I2,11X,I4,11X,F9.3)
     108 FORMAT(1H ,13H R SQUARED = ,F7.5)
         END
$DATA
```

[FORTRAN IV LANGUAGE]

SALES ESTIMATION ANALYSIS - REGRESSION COEFFICIENTS

Problem:

Given regression parameters A, B, C for a PRODUCT

Calculate: m, p, q

where: $M = \frac{-B-\sqrt{B^2-4AC}}{2C}$

$p = A/m$

$q = -mc$

Then:

$$S(T) = \frac{m(p+q)^2}{p} \quad \frac{E^{-(p+q)T}}{[(q/p)\ E^{-(p+q)T}+1]^2}$$

Predicted sales

where T = 1, N — Time periods of perdiction

S_t = Actual sales in time periods — t = 1, n

S(1) = 1st period for which $S_t > A$

$$R^2 = 1 - \frac{\sum_{T=1}^{n} (S(T) - S_t)^2}{\sum_{t=1}^{n} \left(S_t - \frac{\sum \overline{S}_t}{n}\right)^2}$$

Print Output:

1) PARAMETERS M, P, Q and NAME OF PRODUCT

2) S_t Actual Sales, and t, where t goes from 1st sales period where $S_t \geq A$ to n

3) S(T) Predicted Sales, and T time period, with 1 the first sales period where $S_t \geq A$

4) R^2 term

I

IDENTIFICATION CARD

Col.	1	$
	2,3	ID
	4-6	blanks
	7-10	account number assigned by Computer Sci.
	11	*
	12-14	time estimate[1]
	15	*
	16-18	page output estimate[1]
	19-20	**
	21-72	name * any other information

II

CONTROL CARDS[2]

A.	Col.	1	$
		2-8	EXECUTE
		9-15	blanks
		16-20	PUFFET

(or)

B.	Col.	1	$
		2-8	EXECUTE
		9-15	blanks
		16-20	IBJOB
C.	Col.	1	$
		2-6	IBJOB
D.	Col.	1	$
		2-6	IBFTC
		7	blank
		8	SPARCE

---

1 If 8 or less data sets are used time estimate 2 min (002)
page estimate (010)
If 9 or more data sets are used time estimate 5 min (005)
page estimate (050)

2 If 8 or less Data sets are used punch control card A and submit deck under category "P"
If 9 or more data sets are used punch control cards B,C,D and submit deck under category "A".

III

DATA CARDS

A. TITLE CARD

Name of product to be analyzed is punched on this card
Col 1-30 may be used with any data to define name.

B. COEFFICIENTS AND LIMIT CARD

Col.	1-20	VALUE OF COEFFICIENT A
Col.	21-40	VALUE OF COEFFICIENT B
Col.	41-60	VALUE OF COEFFICIENT C
Col.	61-62	MAXIMUM NUMBER OF SALES PERIODS FOR WHICH PREDICTION WILL BE MADE ($N \leq 50$)

Values of A,B,C may be numbers whose total length each is 19 or less digits with 8 or less digits to right of decimal point. Decimal Point must be punched.

N is a two digit number between 01-50. Decimal Point must not be punched.

C. ACTUAL SALES CARD (S)

Col. 1-4 IDENTIFICATION NUMBER FOR SALES VALUE (ALL four digits punched (0001))

Col. 5-14 VALUE OF ACTUAL SALES FOR PARTICULAR IDENTIFICATION NUMBER

Value of Actual sales may have up to 9 digits or less, with 3 or less digits to right of decimal point. Decimal point must be punched.

Col. 80 blank or $

A $ in Col 80 signals the end of a data set. This character must be punched on the Last ACTUAL SALES CARD for the given data set.

The number of ACTUAL SALES CARDS must not exceed 50 and must be less than equal to N (Specified in data card B)

IV

ORDER OF CARDS

1. IDENTIFICATION CARD
2. CONTROL CARD(S)
3. SALES PREDICTION ANALYSIS - PROGRAM DECK

Data Deck:

4. TITLE CARD (A)
5. COEFFICIENTS AND LIMIT CARD (B)
6. ACTUAL SALES CARD(S) (C) [WITH $ IN COL 80 OF LAST CARD]

Items 4-6 may be repeated any number of times if more than one data set is used. See Notes (1) and (2) for proper control cards and job category.

PURDUE UNIVERSITY

KRANNERT SCHOOL OF INDUSTRIAL ADMINISTRATION INSTITUTE PAPER SERIES

(Continued from inside front cover)

105. Michael J. Driver and Siegfried A. Streufert, THE "GENERAL INCONGRUITY ADAPTATION LEVEL" (GIAL) HYPOTHESIS: AN ANALYSIS AND INTEGRATION OF COGNITIVE APPROACHES TO MOTIVATION.
106. William H. Starbuck, THE HETEROSCEDASTIC NORMAL.
107. John J. Sherwood and John R. P. French, SELF-ACTUALIZATION AND SELF-IDENTITY THEORY.*
108. Richard E. Walton and Robert B. McKersie, BEHAVIORAL DILEMMAS IN MIXED MOTIVE DECISION-MAKING.#
109. Stanley Reiter and Donald B. Rice, DISCRETE OPTIMIZING SOLUTION PROCEDURES FOR LINEAR AND NONLINEAR INTEGER PROGRAMMING PROBLEMS.#
110. John J. Sherwood, SELF-REPORT AND PROJECTIVE MEASURES OF ACHIEVEMENT AND AFFILIATION.#*
111. Ronald Kochems, AN APPLICATION OF MULTIPLE DISCRIMINANT ANALYSIS.
112. John A. Shaw, THE THEORY OF CONSUMER RATIONING, PARETO OPTIMALITY, AND THE U.S.S.R.
113. R. K. James, W. H. Starbuck and D. C. King, A STUDY OF PERFORMANCE IN A BUSINESS GAME – REPORT 1.
114. Michael J. Driver, Purdue University, and Siegfried Streufert, Rutgers–The State University, THE GENERAL INCONGRUITY ADAPTATION LEVEL (GIAL) HYPOTHESIS: AN ANALYSIS AND INTEGRATION OF COGNITIVE APPROACHES TO MOTIVATION.
115. Frank M. Bass and Ronald T. Lonsdale, AN EXPLORATION OF LINEAR PROGRAMMING IN MEDIA SELECTION. *
116. Frank M. Bass, THE DYNAMICS OF MARKET SHARE BEHAVIOR.
117. W. H. Starbuck and F. M. Bass, AN EXPERIMENTAL STUDY OF RISK-TAKING AND THE VALUE OF INFORMATION IN A NEW PRODUCT CONTEXT.*
118. John R. P. French, Jr., John J. Sherwood and David L. Bradford, CHANGE IN SELF-IDENTITY IN A MANAGEMENT TRAINING CONFERENCE. #*
119. R. A. Layton, SOME ASPECTS OF THE ECONOMICS OF A COMPUTER SYSTEM STUDY.
120. Walter Sikes, AN ANALYSIS OF SOME OUTCOMES OF HUMAN RELATIONS LABORATORY TRAINING.
121. Charles W. King, COMMUNICATING WITH THE INNOVATOR IN THE FASHION ADOPTION PROCESS. #*
122. R. A. Layton, A "SEARCH AND ESTIMATION" SAMPLING PROCEDURE, WITH APPLICATIONS IN AUDITING AND POVERTY STUDIES.
123. Charles R. Keen, A NOTE ON KONDRATIEFF CYCLES IN PREWAR JAPAN.
124. Robert V. Horton, THE DUALITY IN NATURE OF OFFERINGS OF ADDITIONAL COMMON STOCK BY MEANS OF "RIGHTS".

1966

125. Clarke C. Johnson and Charles E. Gearing, INFLUENCES ON ACADEMIC PERFORMANCE.*
126. Lawrence Carson, Donald Junker, Eugene Rice, Richard Teach, Douglas Tigert, William Urban, EXPERIMENTAL RESEARCH IN CONSUMER BEHAVIOR: FOUR EXPLORATORY PAPERS. *
127. Mohamed A. El-Hodiri, OPTIMAL RESOURCE ALLOCATION OVER TIME I.*
128. Atsushi Suzuki, A LINEAR STATISTICAL MODEL OF AMERICAN BUSINESS CYCLES. *
129. Lowell Bassett, Hamid Habibagahi, James Quirk, QUALITATIVE ECONOMICS AND MORISHIMA MATRICES.*
130. Philip Ginsberg and David Richardson, SOME ECONOMIC APPLICATIONS OF THE GCL PRINCIPLE OF ESTIMATION. *
131. C. S. Yan, OPTIMAL INVESTMENT AND TECHNICAL PROGRESS.
132. C. S. Yan, TECHNICAL CHANGE AND INVESTMENT.
133. Philip Burger and Donald B. Rice, INTEGER PROGRAMMING MODELS OF TRANSPORTATION SYSTEMS – AN AIRLINE SYSTEM EXAMPLE.
134. Mohamed A. El-Hodiri, A CALCULUS PROOF OF THE UNBIASEDNESS OF COMPETITIVE EQUILIBRIUM.
135. Mohamed A. El-Hodiri, TWO ESSAYS ON DYNAMIC MICRO ECONOMICS.
136. Marc Pilisuk, J. Alan Winter, Reuben Chapman, Neil Haas, HONESTY, DECEIT, AND TIMING IN THE DISPLAY OF INTENTIONS.#
137. Richard E. Walton, CONTRASTING DESIGNS FOR PARTICIPATIVE SYSTEMS.
138. Marc Pilisuk, Paul Skolnick, Kenneth Thomas, Reuben Chapman, BOREDOM VS. COGNITIVE REAPPRAISAL IN THE DEVELOPMENT OF COOPERATIVE STRATEGY.#
139. John A. Eisele, Robert Burr Porter, Kenneth C. Young, AN INVESTIGATION OF THE RANDOM WALK HYPOTHESIS AS AN EXPLANATION OF THE BEHAVIOR OF ECONOMIC TIME SERIES.
140. Mogens D. Romer, ELECTRONIC DATA PROCESSING IN INDUSTRIAL ENTERPRISE.
141. Mohamed A. El-Hodiri, CONSTRAINED EXTREMA OF FUNCTIONS OF A FINITE NUMBER OF VARIABLES. REVIEW AND GENERALIZATIONS.
142. Michael J. Driver and Siegfried Streufert, GROUP COMPOSITION, INPUT LOAD AND GROUP INFORMATION PROCESSING.
143. Edgar A. Pessemier and Richard D. Teach, A SINGLE SUBJECT SCALING MODEL USING JUDGED DISTANCES BETWEEN PAIRS OF STIMULI.
144. Harry Schimmler, ON IMPLICATIONS OF PRODUCTIVITY COEFFICIENTS AND EMPIRICAL RATIOS.
145. Hamid Habibagahi, WALRASIAN STABILITY: QUALITATIVE ECONOMICS.

PURDUE UNIVERSITY

KRANNERT SCHOOL OF INDUSTRIAL ADMINISTRATION INSTITUTE PAPER SERIES

(Continued from inside back cover)

146. Edgar A. Pessemier, **MEASURING SOCIAL, SCIENTIFIC AND MILITARY BENEFITS IN A DOLLAR METRIC.**
147. Marc Pilisuk, **DEPTH, CENTRALITY, AND TOLERANCE IN COGNITIVE CONSISTENCY.** #
148. Michael J. Driver and Siegfried Streufert, **THE GENERAL INCONGRUITY ADAPTATION LEVEL (GIAL) HYPOTHESIS – II. INCONGRUITY MOTIVATION TO AFFECT, COGNITION, AND ACTIVATION-AROUSAL THEORY.**
149. Akira Takayama, **BEHAVIOR OF THE FIRM UNDER REGULATORY CONSTRAINT: COMMENT.**
150. Keith V. Smith, **PORTFOLIO REVISION.**
151. Abraham Tesser, Robert D. Gatewood, Michael Driver, **SOME DETERMINANTS OF FEELINGS OF GRATITUDE.**
152. S. N. Afriat, **ECONOMIC TRANSFORMATION.**
153. Edward Ames and Nathan Rosenberg, **THE ENFIELD ARSENAL IN THEORY AND HISTORY.**
154. Robert Perrucci, **HEROES AND HOPELESSNESS IN A TOTAL INSTITUTION: ANOMIE THEORY APPLIED TO A COLLECTIVE DISTURBANCE.**
155. Akira Takayama, **REGIONAL ALLOCATION OF INVESTMENT: A FURTHER ANALYSIS.**
156. Cliff Lloyd, R. J. Rohr and Mark Walker, **A CALCULUS PROOF OF THE EXISTENCE OF A CONTINUOUS UTILITY FUNCTION.**

1967

157. Cliff Lloyd, **MONEY TO SPEND AND MONEY TO HOLD.**
158. Cliff Lloyd, **TWO CLASSICAL MONETARY MODELS.**
159. Robert Perrucci, **SOCIAL PROCESSES IN PSYCHIATRIC DECISIONS.**
160. S. N. Afriat, **PRINCIPLES OF CHOICE AND PREFERENCE.**
161. James M. Holmes, **THE PURCHASING POWER PARITY THEORY: IN DEFENSE OF GUSTAV CASSEL AS A MODERN THEORIST.**
162. John M. Dutton and William H. Starbuck, **HOW CHARLIE ESTIMATES RUN-TIME.**
163. Akira Takayama, **PER CAPITA CONSUMPTION AND GROWTH: A FURTHER ANALYSIS.**
164. Frank DeMeyer and Charles R. Plott, **THE PROBABILITY OF A CYCLICAL MAJORITY.**
165. Siegfried Streufert and Michael J. Driver, **CREATIVITY, COMPLEXITY THEORY AND INCONGRUITY ADAPTATION.**
166. John C. Carlson, **THE CLASSROOM ECONOMY: RULES, RESULTS, REFLECTIONS.**
167. Carl R. Adams, **AN ACTIVITY MODEL OF THE FIRM UNDER RISK.**
168. Charles W. King and John O. Summers, **INTERACTION PATTERNS IN INTERPERSONAL COMMUNICATION.**
169. Vernon L. Smith, **TAXES AND SHARE VALUATION IN COMPETITIVE MARKETS.**
170. James M. Holmes, **AN ECONOMETRIC TEST OF SOME MODERN INTERNATIONAL TRADE THEORIES: CANADA 1870-1960.**
171. Akira Takayama and Mohamed El-Hodiri, **PROGRAMMING, PARETO OPTIMUM AND THE EXISTENCE OF COMPETITIVE EQUILIBRIA.**
172. Marc Pilisuk and Paul Skolnick, **INDUCING TRUST: A TEST OF THE OSGOOD PROPOSAL.**
173. S. N. Afriat, **REGRESSION AND PROJECTION.**
174. Stanley M. Halpin and Marc Pilisuk, **PREDICTION AND CHOICE IN THE PRISONER'S DILEMMA.**